Published by Institute of Physics Publishing for SISSA/ISAS
Received: *June 25, 2002*
Accepted: *August 07, 2002*

An inverse Penrose limit and supersymmetry enhancement in the presence of tensor central charges

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Abstract: A connection between weak and strong tension limits and their perturbative corrections is discussed. New twistor-like models based on $D = 4$, $N = 1$ tensionless superstring and superbrane with tensor central charges are studied. The presence of three, two or less preserved fractions of κ-symmetry in the actions free of the Wess-Zumino terms is shown. A correlation of extra κ-symmetry with the R-symmetry is established. The equations of the superstring and superbrane models preserving 3/4 supersymmetry are exactly solved. The general solution for the Goldstone fermion is pure static, but for the Goldstone bosons it also includes a term describing string/brane motions along the fixed directions given by the initial data. These solutions correspond to the partial spontaneous breaking of the $D = 4$, $N = 1$ global supersymmetry and can be associated with a static closed magnetic Nielsen-Olesen vortex or a p-dimensional vortex.

Keywords: Superstrings and Heterotic Strings, Penrose limit and pp-wave background.

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Contents

1. Introduction

It has recently been shown [1] that the maximally supersymmetric *pp*-wave of IIB superstring [2] can be obtained as a Penrose limit of the supersymmetric $AdS_5 \times S_5$ solution. This ten-dimensional solution is analogous to the maximally supersymmetric solution which preserves 32 supersymmetries of eleven-dimensional supergravity [3]. A physical interpretation of the Penrose limit [4] as corresponding to a large tension (equivalently *weak coupling*) limit of p-brane was suggested in [1]. The proof of this observation was based on the rescaling tension $T_p \to T_p/\Omega^{(p+1)}$, the metric tensor of ten-dimensional supergravity and k-form-field potentials C_k in accordance with the prescription [5] on the Penrose limit extension to the supergravity case. Taking the limit $\Omega^{(p+1)} \to 0$ had resulted to the reduction of brane dynamics to an effective one corresponding to brane embedded into a limiting space. This space depends on the choice of null geodesic used in the perturbative expansion of the brane action fields. As a result the string equations were simplified and exactly solved for the IIB superstring [6]. The resulting IIB superstring theory turned out to be dual to the subsector of $N = 4$ super Yang-Mills theory resulting from the large-$\mathcal{N}$ limit of $\mathrm{U}(\mathcal{N})$ [7]. It was noted [8] that the exact solvability property characterizing the *pp*-wave-like geometry can be broken if the Penrose limit is applied to non-conformal backgrounds.

Let us remind that a perturbative approach using the rescalings of tension, coordinates and fields was earlier considered in [9, 10] for a perturbative solution of non-linear string equations in curved space-time [11]. A generalization of this approach to the case of Dp-branes was considered in [12] with a view of studying the limiting regimes of weak tension and strong coupling [13, 14, 15].

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The dimensionless parameter $\varepsilon = \gamma/\alpha'$ of the rescaled string tension introduced in [9] can be identified with $1/\Omega^2$ from [1]. In [9] the dimensional parameter γ ($[\gamma] = L^2$) was assumed to be a characteristic scale of the curved background geometry. In view of this the parameter ε appeared in the primary string equations for $x^M(\tau,\sigma)$

$$\ddot{x}^M - \left(\frac{\gamma}{\alpha'}\right)^2 x''^M + \Gamma^M_{PQ}(x)\left[\dot{x}^P\dot{x}^Q - \left(\frac{\gamma}{\alpha'}\right)^2 x'^P x'^Q\right] = 0$$

and in one of the primary Virasoro constraints

$$\left(\dot{x}^M G_{MN}\dot{x}^N\right) + \left(\frac{\gamma}{\alpha'}\right)^2 \left(x'^M G_{MN} x'^N\right) = 0\,.$$

Provided that $\gamma/\alpha' \ll 1$ the string equations were considered as non-linear equations with the small parameter ε at the highest derivative with respect to the world-sheet coordinate σ. In the zero approximation these equations were reduced to the equations of a light-like geodesic surface

$$\mathcal{D}_\tau \varphi^M_{,\tau}(\tau,\sigma) = 0 \tag{1.1}$$

describing tensionless string or null string [16]–[19]. The world sheet of null string is a two-dimensional null geodesic manifold, because it consists of a family of the null geodesics $\varphi(\tau,\sigma)$ enumerated by σ and parametrized by the affine parameter τ along each null geodesic. Remind that the generalization to the case of null superstrings, null (super)branes and their quantization were studied in [20, 21, 22]. See also [23]. To study the deformation of null world sheet caused by a weak tension inclusion, the rescaling of the world-sheet coordinate σ

$$\sigma = \varepsilon\xi$$

was done in [9] which was matched with the perturbative expansion for the string world vector

$$x^M(\tau,\sigma) = \varphi^M(\tau,0) + \varepsilon\psi^M(\tau,\xi) + \varepsilon^2\chi^M(\tau,\xi) + \cdots\,.$$

This procedure is a mathematical one for fixing the restriction to study the world-sheet deformations in the vicinity of its geodesic line, e.g. $\varphi^M(\tau,\sigma)|_{\sigma=0}$, representing the congruent family of null geodesics forming the null-string world sheet. In the first approximation a small deformation of the null world sheet is described by the function $\psi^M(\tau,\xi)$ defined by the linear perturbative equation

$$\left(\mathcal{D}^2_\tau - \partial^2_\xi\right)\psi^M - R^M_{PQL}\varphi^P_{,\tau}\varphi^Q_{,\tau}\psi^L = 0\,,$$

where R^M_{PQL} is the Riemann-Christoffel tensor, and the constraints [9, 10]

$$\left(\varphi_{M,\tau}\mathcal{D}_\tau\psi^M\right) = 0\,, \qquad \left(\varphi_{M,\tau}\psi^M\right) = 0\,.$$

The equation for ψ^M is the general covariant extension of the geodesic deviation eq. [24] by the oscillatory term $\partial^2_\xi\psi^M$. The term $\partial^2_\xi\psi^M$ signals on the appearance of tension and

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encodes the effect of the string elastic force pushing out the null string points from the null geodesics. For the class of symmetric spaces characterized by the condition

$$R_{MPQL} = \kappa(G_{MQ}G_{PL} - G_{ML}G_{PQ})$$

the equation for ψ^M acquires a simple form of the general covariant wave equation

$$\left(\mathcal{D}_\tau^2 - \partial_\xi^2\right)\psi^M = 0\,.$$

The equation was reduced to a linear system of modified Bessel equations for the de Sitter and Friedmann-Robertson-Walker spaces and it was exactly solved [10]. As a result, the string equations in the de Sitter space can also be quantized in a first approximation in the rescaled tension parameter ε or, equivalently, for a large Hubble constant H ($\alpha' H^2 \gg 1$), if the parameter γ is identified with $1/H^2$.

The limit of strong tension $\Omega^2 \to 0$ [2], or equivalently $\varepsilon = \gamma/\alpha' \to \infty$, is the inverse limit to the above-considered one named here as an inverse Penrose limit. Thus, it seems to be expedient to apply the Penrose limiting procedure [4] directly inside the primary string equations and constraints, because they are correct at any value of the parameter ε. It can be achieved by the introduction of local coordinates in a neighbourhood of a null geodesic φ^M belonging to the above considered geodesic null worldsheet. After the introduction of these local coordinates we must perform their rescaling accompanied with the transformation of the background metric $g_{MN} \to g_{MN}(1/\varepsilon)$ following the Penrose prescription. At last, the $\lim\limits_{\varepsilon\to\infty}[\varepsilon g_{MN}(1/\varepsilon)]$ is to be considered in the transformed string equations and constraints. This limit should give the correct description of string dynamics in the limiting pp-wave background complemented by perturbative corrections deforming the zero pp-wave approximation of the original background. Then there arises the possibility to compare the strong and weak perturbative expansions in the vicinity of the null geodesic φ^M. So, one can try to formulate the transformations connecting these perturbative solutions to get a new information on the structure and symmetries of non-perturbative string solutions.

Study of this problem is strongly motivated by the case of Dp-brane theory, where a strong coupling limit of the generalized Born-Infeld action and the two-form $\mathcal{F} = B + (\gamma/2\pi\alpha')^{-1}F$ were studied [12, 25] and a connection between tensionless strings/branes, non-commutative geometry [26]–[29] and NCOS theory [30, 31, 32] was observed. In particular, the electric field E of tensionless Dp-brane was shown to become constant $E = \gamma/\pi\alpha'$ (if $B = 0$) and to coincide with the critical value of electric field [33] given by the NCOS theory. Investigation of the correlations between the space-time structure in M theory and possible mechanisms to generate string/brane tension deserves to be continued.

A duality connection between the strong and weak tension limits hints to a possibility of the supersymmetry enhancement in the models based on tensionless superstrings and superbranes. This problem can surely be studied in the string/brane model with the $D = 4$, $N = 1$ supersymmetry enlarged by the tensor central charges (TCC) Z_{mn} describing a brane/string contribution into the supersymmetry algebra. An interesting example of 1/2 supersymmetric configurations encoded by the so-enlarged supesymmetry algebra is the domain wall [34, 35]. Similar domain wall is also created by the gluino condensate in the

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SU(n) QCD [36, 37] due to the spontaneous breakdown of discrete chiral symmetry [38]. A wide class of interesting string/brane models considering the TCC coordinates $z_{m_1m_2...m_p}$ as new independent degrees of freedom associated with the p-form generators $Z_{m_1m_2...m_p}$ was studied in [39]–[45].

The existence of BPS states preserving $1/4, 1/2, 3/4$ of the $D=4$, $N=1$ supersymmetry enlarged by the TCC Z_{mn} was proved in [46] using a model-independent analysis of the enlarged supersymmetry algebra. The connection of these BPS states with the Jordan algebra of 4×4 real symmetric matrices and associated geometric structures was revealed in [46], where the combinations of momentum and domain-wall charges corresponding to the BPS states were found. Realizations of supersymmetry configurations with enhanced supersymmetry (such as 3/4 supersymmetry) in string/M-theory were studied in [47, 48, 49]. The superparticle model with 3/4 supersymmetry in the enlarged space-time was constructed in [50, 51]. A general class of pp-wave solutions preserving fractions ν, with $1/2 < \nu < 1$, of the supersymmetry in the type-IIB theory and M-theory was described in [52, 53]. A pp-wave of M-theory preserving 3/4 supersymmetry was realized in [54]. In [55] the solutions of M-theory with 18, 20, 22 and 24 extra supersymmetries were presented. General algebraic and geometric foundations for the enlargement of superspace by adding tensor central charge coordinates connected with the free differential algebra ideas [56] were developed in [57].

To clear up the dynamical role of TCC coordinates, a new class of string models in the $D=4$ space-time enlarged by the six real TCC coordinates z_{mn} was constructed in [25] as a natural generalization of the twistor-like formulations [58] for the Nambu-Goto and Schild string actions. Note that twistor formulation of the action [58] was given in [59] based on a twistor description of null two-surfaces similar to that of [60]. Twistor approach to supersymmetry was studied in [61]–[64]. In [25] the twistor-like model of tensionless string minimally extended by the term linear in the derivatives of TCC coordinates was shown to be exactly solvable. It was established that the inclusion of z^{mn} lifts the light-like character of the tensionless string worldsheet and removes the degeneracy of the worldsheet metric. Study of the hamiltonian structure and symmetries of the minimally extended model shows that the string constraints reduce the number of independent TCC coordinates z_{mn} to one real effective coordinate [65]. This corresponds to the string moving in an effective (4+1)-dimensional target space instead of the original (4+6)-dimensional space-time. The P.B. algebra for the first-class constraints of the model has a structure similar to that of the contracted algebra of rotations of an (Anti) de Sitter space [66], but the second-class constraints have to deform the P.B. algebra into the Dirac bracket algebra. The Lorentz-covariant antisymmetric 9×9 complex Dirac $\hat{\mathbb{C}}$-matrix of the P.B. of the second-class constraints has a rather clear algebraic structure [65], and the D.B. algebra construction of the first-class constraints will shed light on the structure of the effective five-dimensional target-space connected with the TCC. It seems reliable to connect this space with the appropriate boundary of the convex cone analysed in [46].

An effective role of TCC in the transition from ten-dimensional Minkowski space-time to $AdS_5\times S_5$ was observed in [67, 68]. The connections between TCC, symplectic superalgebras and the theory of massless higher spin fields and currents were discussed

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in [69]. So, one may suppose that the twistor-like actions [58, 65], constructed from very simple geometric objects in the space-time enlarged by TCC coordinates and auxiliary twistor-like variables can code a hidden dynamics relevant to that in the backgrounds with an enhanced supersymmetry.

In this paper we study supersymmetric generalizations of the twistor-like string models with TCC [58, 65] and propose new models in the enlarged superspace which preserve $3/4, 1/2$ or smaller fractions of $D = 4$, $N = 1$ global supersymmetry. A basic role of the presence of TCC coordinates for the supersymmetry enhancement is clarified. We show that the superstring and superbrane models preserving 3/4 supersymmetry are exactly solvable, and find their general solutions which correspond to partially broken $D = 4$, $N = 1$ global supesymmetry. The general solution for the Goldstone fermion [70]–[73] is a pure static one. The solution for the Goldstone bosons includes a pure static term and a term describing string/brane motions along fixed directions given by the initial data of the considered variational problem. These solutions can be associated with a static closed spin or magnetic Nielsen-Olesen vortex [74]. We extend these results to the case of some exactly solvable super p-brane models generalizing the models of tensionless p-branes and associate their Goldstone solutions with p-dimensional magnetic vortices.

In section 2 we formulate the transformation rules for the coordinates of the enlarged superspace with respect to the global supersymmetry and κ-symmetry. The enlarged 4×4 matrix of the Cartan one-forms W_{ab} symmetric in the Majorana spinor indices a, b is presented and its κ-symmetry transformation rules are defined.

In section 3 the matrix elements of the matrix one-form W_{ab} in the linear space of local Newman-Penrose dyads are studied, and the conditions for their invariance under the κ-symmetry are formulated. We show that the maximum supersymmetry enhancement, preserving 3/4 supersymmetry, takes place for the diagonal matrix elements only.

In section 4 we connect the maximum enhancement of the supersymmetry with the presence of the R-symmetry in the diagonal Cartan one-forms.

In section 5 these scalar diagonal one-forms are used to construct a new model of the $D = 4$, $N = 1$ superstring preserving 3/4 supersymmetry. The exact solutions of the model are presented there.

In section 6 the above-discussed results are generalized to the case of super p-branes, and their actions preserving 3/4 supersymmetry are presented. We show that the general solutions for the Goldstone fermion and bosons are similar to the superstring ones, but additional constraints for the initial data appear here.

Finally, section 7 contains conclusions and discussion concerning possible ways of application of the obtained results.

2. Supersymmetry, κ-symmetry and the Newman-Penrose dyads

It is well known that the inclusion of the tensor central charge (TCC) two-form Z_{mn} modifies the anticommutator of the $D = 4$, $N = 1$ Majorana supercharges Q_a

$$\{Q_a, Q_b\} = \left(\gamma^m C^{-1}\right)_{ab} P_m + i \left(\gamma^{mn} C^{-1}\right)_{ab} Z_{mn} \,. \tag{2.1}$$

The real parameters z_{mn} corresponding to Z_{mn} may be presented in the spinor form

$$z_{ab} = iz_{mn}\left(\gamma^{mn}\, C^{-1}\right)_{ab}\,, \tag{2.2}$$

by analogy with the spinor representation for the space-time coordinates x_m

$$x_{ab} = x_m(\gamma^m\, C^{-1})_{ab}\,. \tag{2.3}$$

The coordinates z_{ab} and x_{ab} may be treated as the components of the real symmetric spin-tensor

$$Y_{ab} \equiv x_{ab} + z_{ab} \tag{2.4}$$

associated with the 4×4 spinor matrix $Y_a{}^b$

$$Y_a{}^b = Y_{ad}C^{db} = \begin{pmatrix} z_\alpha{}^\beta & x_{\alpha\dot\beta} \\ \tilde{x}^{\dot\alpha\beta} & \bar{z}^{\dot\alpha}{}_{\dot\beta} \end{pmatrix}, \qquad z_\alpha{}^\beta = \varepsilon_{\alpha\gamma}z^{\gamma\beta}, \tag{2.5}$$

where C is the charge conjugation matrix

$$C^{ab} = \begin{pmatrix} \varepsilon^{\alpha\beta} & 0 \\ 0 & \varepsilon_{\dot\alpha\dot\beta} \end{pmatrix} \tag{2.6}$$

chosen to be imaginary in the Majorana representation similarly to the γ^m-matrices. The matrix $Y_a{}^b$ (2.5) paired with the Grassmann Majorana spinor θ_a

$$\theta_a = \begin{pmatrix} \theta_\alpha \\ \bar\theta^{\dot\alpha} \end{pmatrix}, \qquad \theta^a = C^{ab}\theta_b\,, \qquad \theta^\alpha = \varepsilon^{\alpha\beta}\theta_\beta \tag{2.7}$$

is a compact generalized coordinate convenient to construct superstring models in the enlarged superspace (Y_{ab},θ_a), because the $D=4N=1$ global supersymmetry transforms the superspace (Y_{ab},θ_a) into itself

$$\delta_\varepsilon\theta_a = \varepsilon_a\,, \qquad \delta_\varepsilon Y_{ab} = 2i(\theta_a\varepsilon_b + \theta_b\varepsilon_a)\,. \tag{2.8}$$

The Cartan differential one-forms W_a and W_{ab} invariant under the transformations (2.8) are given by

$$W_a = d\theta_a\,, \qquad W_{ab} = dY_{ab} - 2i(d\theta_a\theta_b + d\theta_b\theta_a)\,. \tag{2.9}$$

A realization of κ-symmetry transformations may be chosen in the form

$$\delta_\kappa\theta_a = \kappa_a\,, \qquad \delta_\kappa Y_{ab} = -2i(\theta_a\kappa_b + \theta_b\kappa_a)\,, \tag{2.10}$$

where $\kappa_a(\tau,\sigma)$ is a Majorana spinor. Using (2.9) and (2.10) we find the transformation rules for the supersymmetric Cartan one-forms under the κ-symmetry

$$\delta_\kappa W_a = d\kappa_a\,, \qquad \delta_\kappa W_{ab} = -4i(d\theta_a\kappa_b + d\theta_b\kappa_a)\,. \tag{2.11}$$

Following the approach studied in [25] we intend here to extend the (Y_{ab},θ_a)-superspace by the addition of the local Newman-Penrose dyads u^α, v_α attached to the superstring worldsheet. The dyads are suitable twistor-like variables defined by their properties

$$u^\alpha v_\alpha \equiv u^\alpha\varepsilon_{\alpha\beta}v^\beta = 1\,, \qquad u^\alpha u_\alpha = v^\alpha v_\alpha = 0 \tag{2.12}$$

and may be treated as the Weyl components of the Majorana spinors U_a and V_a

$$U_a = \begin{pmatrix} u_\alpha \\ \bar{u}^{\dot\alpha} \end{pmatrix}, \; V_a = \begin{pmatrix} v_\alpha \\ \bar{v}^{\dot\alpha} \end{pmatrix}, \qquad \left(U^a \gamma_{5a}{}^b V_b\right) = -2i, \tag{2.13}$$

where $(\gamma_5)_a{}^b$ is the diagonal matrix in the Weyl representation

$$(\gamma_5)_a{}^b = \begin{pmatrix} -i\delta_\alpha{}^\beta & 0 \\ 0 & i\delta^{\dot\alpha}{}_{\dot\beta} \end{pmatrix}. \tag{2.14}$$

Applying the ideas advanced in [75] (see also [76, 77]) we assume here that the dyads U_a and V_a are invariants of the transformations (2.8) and (2.10)

$$\delta_\varepsilon U_a = \delta_\varepsilon V_a = 0\,, \qquad \delta_\kappa U_a = \delta_\kappa V_a = 0\,. \tag{2.15}$$

Having fixed the transformation rules for the fields Y_{ab}, θ_a, U_a and V_a one can proceed to the construction of new κ-invariant actions for superstrings with the TCC coordinates.

3. κ-symmetry and invariant one-forms

Let us consider the one-form W_{ab} (2.9) and construct the following supersymmetric and Lorentz invariant set of one-forms

$$W^{(IJ)}_{\varphi\chi} = \varphi_a (IWJ)^{ab}\chi_b\,, \tag{3.1}$$

where the Majorana spinors φ_a and χ_a are the linear combinations of U_a, V_a and $(\gamma_5 U)_a$, $(\gamma_5 V)_a$ forming the representation of the dyad space (2.12) and (2.13) by the Majorana spinors

$$\begin{aligned} \varphi_a &= \left[\left(\varphi^{(u)}_R - \varphi^{(u)}_I\gamma_5\right)U + \left(\varphi^{(v)}_R - \varphi^{(v)}_I\gamma_5\right)V\right]_a, \\ \chi_a &= \left[\left(\chi^{(u)}_R - \chi^{(u)}_I\gamma_5\right)U + \left(\chi^{(v)}_R - \chi^{(v)}_I\gamma_5\right)V\right]_a \end{aligned} \tag{3.2}$$

and I^{ab}, J^{ab} are presented by the antisymmetric γ-matrices

$$I^{ab}, J^{ab} = \left\{C^{ab},\, (C\gamma_5)^{ab},\, p^m(C\gamma_m\gamma_5)^{ab}\right\}, \tag{3.3}$$

where p^m is a κ-invariant vector or pseudovector in the $D = 4$ space-time

$$p_m = (\tilde\varphi C\gamma_m\gamma_5\tilde\chi)\,, \qquad \text{or} \qquad (\tilde\varphi C\gamma_m\tilde\chi) \tag{3.4}$$

constructed of the Majorana spinors $\tilde\varphi$, $\tilde\chi$ selected from the dyad space (3.2) invariant under the κ-symmetry (2.15). The analysis of the case when I^{ab}, J^{ab} are symmetric matrices

$$I^{ab}, J^{ab} = \left\{\pi^m(C\gamma_m)^{ab},\, \pi^{mn}(C\gamma_{mn})^{ab}\right\} \tag{3.5}$$

just as the case of I and J belonging to the various sets is reduced to the analysis of the case (3.3). Thereat using (2.11) and (2.15) we find the κ-transformation of $W^{(IJ)}_{\varphi\chi}$

$$\delta_\kappa W^{(IJ)}_{\varphi\chi} = -4i[(\varphi I d\theta)(\kappa J\chi) - (\varphi I\kappa)(d\theta J\chi)] \tag{3.6}$$

and conclude that the κ-invariance of $W^{(IJ)}_{\varphi\chi}$ allows some off-shell restrictions for the κ-symmetry parameter κ_a. Note the change of the sign inside the square brackets (3.6) for the case of I and J belonging to the various matrix sets. For the antisymmsetric set (3.3) these restrictions are subdivided into three cases A, B and C.

For the A-case I^{ab} and J^{ab} take their values from the shortened subset (3.3)

$$I', J' = (C,\, C\gamma_5) \tag{3.7}$$

and we obtain the following two real off-shell conditions for the four real components κ_a

$$\big(\kappa J'\chi\big) = 0\,, \qquad \big(\kappa I'\varphi\big) = 0 \tag{3.8}$$

providing partial κ-invariance of $W^{(I'J')}_{\varphi\chi}$ under 1/2 fraction of the full κ-symmetry (2.10).

The B-case corresponds to the choice

$$I = I' = (C,\, C\gamma_5)\,, \qquad J = p^m(C\gamma_5\gamma_m) \tag{3.9}$$

or vice versa. It is easy to see that any matrix Γ_A from the complete set of the sixteen γ-matrices when multiplied by p_A maps the dyad space onto itself

$$\big(p^A\Gamma_A\big)\,\varphi = \tilde{\varphi}\,, \qquad p^A \equiv \big(\lambda\Gamma^A\psi\big) \tag{3.10}$$

including the case $\tilde{\varphi}$=0. This property of the dyad space follows from the completeness of the Pauli matrices $\sigma_{m\alpha\dot{\alpha}}\tilde{\sigma}^{m\dot{\beta}\beta} = -2\delta^{\beta}_{\alpha}\,\delta^{\dot{\beta}}_{\dot{\alpha}}$ and the contraction conditions (2.12). Using this observation we find that the B-case is reduced to the A-case yielding two real conditions for the components of κ_a. The same conclusion remains valid for the C-case

$$I = p^m(C\gamma_5\gamma_m)\,, \qquad J = \tilde{p}^m(C\gamma_5\gamma_m)\,, \tag{3.11}$$

corresponding to the one-form $W^{(C\gamma_5 p^m\gamma_m, C\gamma_5\tilde{p}^m\gamma_m)}_{\varphi\chi}$. We conclude that all the one-forms (3.1) containing different spinors from the dyad space (3.2) are invariant under 1/2 κ-symmetry. Therefore, without loss of generality one can be restricted to studying the invariant one-forms which belong to the set (3.7).

Now we observe the certain case of the symmetry enhancement when one extra κ-symmetry appears. This case corresponds to the choice $\varphi = \chi$ and $I = J$, or equivalently to

$$I' = J' = (C,\;\, C\gamma_5)\,, \qquad \varphi = \chi\,, \tag{3.12}$$

because then both of the conditions (3.8) coincide giving one real condition

$$(\kappa J'\varphi) = 0\,. \tag{3.13}$$

As a result, each of the one-forms W_φ

$$W_\varphi \equiv W^{(C,C)}_{\varphi\varphi} = \varphi_a W^{ab}\varphi_b\,, \qquad (\kappa C\varphi) = 0 \tag{3.14}$$

and $W_{\gamma_5\varphi}$

$$W_{\gamma_5\varphi} \equiv W^{(C\gamma_5,\, C\gamma_5)}_{\varphi\varphi} = (\gamma_5\varphi)_a W^{ab}(\gamma_5\varphi)_b\,, \qquad (\kappa C\gamma_5\varphi) = 0 \tag{3.15}$$

becomes invariant under the corresponding three parametric κ-symmetry. Thus, we arrive to the one-forms preserving 3/4 κ-symmetry in the superspace $(Y_{ab}, \theta_a, u_\alpha, v_\alpha)$.

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To clarify this result we note that the k-variation (3.6) takes the form

$$\delta_\kappa W_\varphi|_{\omega_{\alpha\beta}=0} = 4i\left[(\varphi^\alpha d\theta_\alpha)\left(\bar{\kappa}_{\dot\beta}\bar{\varphi}^{\dot\beta}\right) - \left(\kappa_\beta\varphi^\beta\right)\left(\bar{\varphi}^{\dot\alpha}d\bar{\theta}_{\dot\alpha}\right)\right] \tag{3.16}$$

in the absence of the TCC coordinates, as it follows from the component representation

$$W_\varphi = 2\varphi^\alpha\omega_{\alpha\dot\alpha}\bar{\varphi}^{\dot\alpha} + \varphi^\alpha\omega_\alpha{}^\beta\varphi_\beta + \bar{\varphi}_{\dot\alpha}\bar{\omega}^{\dot\alpha}{}_{\dot\beta}\bar{\varphi}^{\dot\beta}\,. \tag{3.17}$$

Equation (3.16) shows that κ-invariance of $W_\varphi|_{\omega_{\alpha\beta}=0}$ implies two real off-shell conditions

$$\mathrm{Im}(\kappa_\alpha\varphi^\alpha) = 0\,, \qquad Re(\kappa_\alpha\varphi^\alpha) = 0 \tag{3.18}$$

instead of the one real condition (3.13)

$$\mathrm{Im}(\kappa_\alpha\varphi^\alpha) = 0\,. \tag{3.19}$$

So, we reveal the cancellation of the contributions into the real part of $(\kappa_\alpha\varphi^\alpha)$ given by the one-form $\omega_{\alpha\dot\alpha}$ connected with x_m and the one-forms $\omega_\alpha{}^\beta$, $\bar{\omega}^{\dot\alpha}{}_{\dot\beta}$ connected with the TCC coordinates z_{mn}. Analogously, the κ-invariance of W_φ in the absence of the space-time coordinates x^m implies the same restrictions (3.18) due to the relation

$$\delta_\kappa W_\varphi|_{\omega_{\alpha\dot\beta}=0} = -8i\left[(\varphi^\alpha d\theta_\alpha)\left(\kappa_\beta\varphi^\beta\right) - \left(\bar{\kappa}_{\dot\beta}\bar{\varphi}^{\dot\beta}\right)\left(\bar{\varphi}^{\dot\alpha}d\bar{\theta}_{\dot\alpha}\right)\right]. \tag{3.20}$$

Next we observe that the opposite case of the total κ-symmetry breakdown may be realized by consideration of a linear combination of the primary and partially κ-invariant one-forms (3.1) or by using p_A which is not a κ-invariant object.

Therefore, we see that the appearance of the third extra κ-symmetry is a collective effect of the addition of z_{mn} and of only one spinor from the dyad space. This result can be understood from the point of view of symmetry, because the third κ-symmetry is accompanied with the R-symmetry. We shall discuss this observation in the next section.

4. 3/4 κ-symmetry and the R-symmetry

The R-symmetry is defined by the U(1) transformations of the Weyl θ-spinor

$$\theta'_\alpha = e^{-ia_R}\theta_\alpha\,, \qquad \bar{\theta}'_{\dot\alpha} = e^{ia_R}\bar{\theta}_{\dot\alpha}\,, \tag{4.1}$$

where a_R is a real parameter. Alternatively, (4.1) may be presented as an axial rotation

$$\theta'_a = \begin{pmatrix} e^{-ia_R}\theta_\alpha \\ e^{ia_R}\bar{\theta}^{\dot\alpha} \end{pmatrix} = (e^{a_R\gamma_5}\theta)_a \tag{4.2}$$

of the Majorana bispinor θ_a using the relation $e^{a_R\gamma_5} = cosa_R + \gamma_5 sina_R$.

The anticommutation relation (2.1)

$$\{Q_\alpha, Q_\beta\} = Z_{\alpha\beta}\,, \qquad \left\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\right\} = \bar{Z}_{\dot\alpha\dot\beta} \tag{4.3}$$

fixes the transformation rules for TCC coordinates under the R-symmetry (4.1)

$$z'_{\alpha\beta} = e^{-2ia_R} z_{\alpha\beta} , \qquad \bar{z}'_{\dot\alpha\dot\beta} = e^{2ia_R} \bar{z}_{\dot\alpha\dot\beta} . \tag{4.4}$$

In the Majorana representation the axial rotation (4.2) can be rewritten as

$$z'^{\ b}_{a} = (e^{a_R\gamma_5} z e^{a_R\gamma_5})_a{}^b . \tag{4.5}$$

As a consequence of (4.2) and (4.5), we find the R-transformations of $Y_a{}^b$ (2.5) and $W_a{}^b$ (2.9)

$$\begin{aligned} Y'^{b}_{a} &= (e^{a_R\gamma_5} Y e^{a_R\gamma_5})_a{}^b , \\ W'^{b}_{a} &= (e^{a_R\gamma_5} W e^{a_R\gamma_5})_a{}^b . \end{aligned} \tag{4.6}$$

As a result of (4.6), the R-transformation of the one-form W_φ is given by

$$W'_\varphi \equiv (\varphi W' \varphi) = (\varphi' W \varphi') , \tag{4.7}$$

where the transformed spinor φ' is defined as

$$\varphi' = e^{a_R\gamma_5} \varphi . \tag{4.8}$$

Right now we observe that the γ_5-transformation (4.8) belongs to the local group $[\mathrm{U}(1) \times \mathrm{O}(1,1)]_{\mathrm{dyad}}$ transforming the base dyad spinors U_a and V_a

$$\begin{aligned} U' &= e^{-(\alpha_I + \alpha_R\gamma_5)} U , \\ V' &= e^{(\alpha_I + \alpha_R\gamma_5)} V . \end{aligned} \tag{4.9}$$

The gauge group $[\mathrm{U}(1) \times \mathrm{O}(1,1)]_{\mathrm{dyad}}$ is the symmetry group of the relations (2.12) which defines the Newman-Penrose dyads

$$u'_\alpha = e^{i\alpha} u_\alpha , \qquad v'_\alpha = e^{-i\alpha} v_\alpha , \tag{4.10}$$

where α is a complex parameter

$$\alpha = \alpha_R + i\alpha_I . \tag{4.11}$$

Without loss of generality one can choose φ in (4.7) and (4.8) to be equal to U (or V), thus making evident the fact that the axial rotation (4.8) is compensated by the $[\mathrm{U}(1) \times \mathrm{O}(1,1)]_{\mathrm{dyad}}$ transformations (4.9) defined by

$$\alpha_R = a_R , \qquad \alpha_I = 0 . \tag{4.12}$$

Due to this compensation we may consider the one-form W_U (3.14) as an effective invariant of the R-symmetry. On the other hand, the analogous compensating $[\mathrm{U}(1) \times \mathrm{O}(1,1)]_{\mathrm{dyad}}$ transformation for the one-form W_V (3.14) is defined by the conditions

$$\alpha_R = -a_R , \qquad \alpha_I = 0 \tag{4.13}$$

as it follows from (4.8). We see, therefore, that the conditions (4.12) and (4.13) are mutually exclusive, and W_V breaks the R-symmetry if W_U preserves it.

In addition we observe that the compensating transformation (4.12) also allows to consider the product $(\kappa_a U^a)$ as an effective invariant of the R-symmetry and the constraint (3.13)

$$(\kappa_a \varphi^a)|_{\varphi=U} = 0 \tag{4.14}$$

does not break the R-symmetry. In that case, however, the product $(\kappa_a V^a)$ is not invariant of the R-symmetry and the condition (3.13)

$$(\kappa_a \varphi^a)|_{\varphi=V} = 0 \tag{4.15}$$

implies the R-symmetry breaking. We see that the conditions of the presence of the R-symmetry are the same as those for the presence of extra κ-symmetry and the mutual presence of the conditions (4.14) and (4.15) breaks this extra κ-symmetry.

Thus, we have fixed the correlation between the extra κ-symmetry and the R-symmetry. The one-form W_φ invariant under 3/4 κ-symmetry and the R-symmetry will be used to build a superstring action preserving 3/4 supersymmetry.

5. A superstring model with 3/4 k-symmetry

An example of the superstring action with extra κ-symmetry may be formulated in terms of the invariant one-form W_φ discussed in the previous section

$$S_\varphi = \frac{k}{2}\int d\tau d\sigma \varrho^\mu \left(\varphi_a W^{ab}_\mu \varphi_b\right), \tag{5.1}$$

where W^{ab}_μ is the world-sheet pullback of the one-form W^{ab}

$$W^{ab}_\mu \equiv \partial_\mu Y^{ab} + 2i\left(\partial_\mu \theta^a \theta^b + \partial_\mu \theta^b \theta^a\right), \qquad (\mu = (\tau, \sigma)) \tag{5.2}$$

and $\varrho^\mu(\tau,\sigma)$ is a κ-invariant world-sheet density which provides the reparametrization invariance of S_φ analogously to the cases of tensionless superbrane [22] or the Green-Schwarz superstring [78]. The dimensional constant k in (5.2) can be moved in a redefinition of x_m and z_{mn} making all dynamical variables dimensionless. We consider a closed superstring and fix φ_a

$$\varphi_a = U_a\,. \tag{5.3}$$

Then the action (5.2) transforms into the action

$$S_U = \frac{1}{2}\int d\tau d\sigma \varrho^\mu \left(U_a W^{ab}_\mu U_b\right) \tag{5.4}$$

invariant (by construction) under the three parametric κ-symmetry (2.10)

$$\delta_\kappa \varrho^\mu = 0\,, \qquad (\kappa_a U^a) = 0\,. \tag{5.5}$$

The variations of S_U with respect to ϱ_μ, U^b, Y_{ab} and θ_b give the equations of motion

$$U^a[\partial_\mu Y_{ab} - 2i(\partial_\mu \theta_a \theta_b + \partial_\mu \theta_b \theta_a)]U^b = 0\,, \tag{5.6}$$

$$\varrho^\mu[\partial_\mu Y_{ab} - 2i(\partial_\mu \theta_a \theta_b + \partial_\mu \theta_b \theta_a)]U^b = 0\,, \tag{5.7}$$

$$\partial_\mu\left(\varrho^\mu U^a U^b\right) = 0\,, \tag{5.8}$$

$$(\varrho^\mu \partial_\mu \theta_b)U^b = 0\,. \tag{5.9}$$

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In view of 3/4 κ-symmetry of S_U we can fix its gauge by the conditions

$$(V^a\theta_a) = 0\,, \qquad (V^a\gamma_{5a}{}^b\theta_b) = 0\,, \qquad (U^a\gamma_{5a}{}^b\theta_b) = 0 \tag{5.10}$$

and introduce the κ-invariant Grassmann variable η

$$\eta \equiv \frac{1}{2i}(U^a\theta_a) \tag{5.11}$$

which incodes the rest of the dynamical degrees of freedom θ^a and describes the Goldstone fermion. Next let us take into account the dyad expansion of θ_α

$$\begin{aligned}\theta_\alpha &= \theta^{(u)}u_\alpha + \theta^{(v)}v_\alpha\,,\\ \bar{\theta}_{\dot{\alpha}} &= \bar{\theta}^{(u)}\bar{u}_{\dot{\alpha}} + \bar{\theta}^{(v)}\bar{v}_{\dot{\alpha}}\end{aligned} \tag{5.12}$$

which may be presented in terms of the Majorana spinors as

$$\theta_a = \left[\left(\theta_R^{(u)} - \theta_I^{(u)}\gamma_5\right)U + \left(\theta_R^{(v)} - \theta_I^{(v)}\gamma_5\right)V\right]_a. \tag{5.13}$$

With the help of (5.13) we find the following representation

$$\theta_a = -\theta_I^{(v)}(\gamma_5 V)_a = -\eta(\gamma_5 V)_a. \tag{5.14}$$

for θ_a as a function of η. To solve eqs. (5.6)–(5.9) we choose an additional gauge fixing

$$\varrho^\sigma(\tau,\sigma) = 0 \tag{5.15}$$

allowed by the reparametrization invariance of S_U. Then eq. (5.9) reduces to

$$2i\dot{\eta} + (\dot{U}\gamma_5 V)\eta = 0 \tag{5.16}$$

and the substitution

$$\eta = e^{\Lambda(\tau,\sigma)}\eta_0(\sigma)\,, \qquad (\eta_0)^2 = 0 \tag{5.17}$$

transforms (5.16) into the equation

$$2i\dot{\Lambda} + (\dot{U}\gamma_5 V) = 0\,. \tag{5.18}$$

In the gauges (5.10) and (5.15) eqs. (5.6), (5.7) and (5.8) take the form

$$U^aY'_{ab}U^b - 16i\eta\eta' = 0\,, \tag{5.19}$$

$$\dot{Y}_{ab}U_b = 0\,, \tag{5.20}$$

$$(\varrho^\tau U_aU_b)^{\cdot} = 0\,, \tag{5.21}$$

where the relation

$$\eta\dot{\eta} = 0 \tag{5.22}$$

resulting from (5.17) has been used. Equation (5.21) multiplied by $(\gamma_5 V)^a$ gives

$$\varrho^\tau\dot{U}^a = \left[-\dot{\varrho}^\tau + \frac{1}{2i}\varrho^\tau\left(\dot{U}\gamma_5 V\right)\right]U^a \tag{5.23}$$

and the eq. (5.23) can be written as

$$\varrho^\tau \dot{U}^a = -\left[\dot{\varrho}^\tau + \varrho^\tau \dot{\Lambda}\right] U^a \tag{5.24}$$

after using eq. (5.18). The general solution of eq. (5.24) is

$$U^a(\tau,\sigma) = \frac{1}{\varrho^\tau} e^{-\Lambda(\tau,\sigma)} U_0^a(\sigma) \,. \tag{5.25}$$

The substitution of (5.25) into eqs. (5.18) and (5.21) gives

$$2\dot{\Lambda} + \frac{\dot{\varrho}^\tau}{\varrho^\tau} = 0 \tag{5.26}$$

and its solution is

$$\Lambda(\tau,\sigma) = -\frac{1}{2} \ln\left[\varrho^\tau \lambda_0(\sigma)\right] , \tag{5.27}$$

where $\lambda_0(\sigma)$ is an arbitrary function.

Using (5.27) one can present the general solutions of eqs. (5.16) and (5.24) as

$$\begin{aligned} \eta(\tau,\sigma) &= \frac{1}{\sqrt{\varrho^\tau}} \eta_0(\sigma) \,, \\ U^a(\tau,\sigma) &= \frac{1}{\sqrt{\varrho^\tau}} U_0^a(\sigma) \,, \end{aligned} \tag{5.28}$$

where $\lambda_0(\sigma)$ is contained in the redefinition of ϱ^τ and $U_0^a(\sigma)$. The substitution of U^a (5.28) into (2.13) and of η into (5.14) gives the following solutions for V^a and θ_a

$$\begin{aligned} V^a(\tau,\sigma) &= \sqrt{\varrho^\tau}\, V_0^a(\sigma) \,, \\ \theta_a \equiv \theta_a(\sigma) &= -\eta_0(\sigma)(\gamma_5 V_0(\sigma))_a \end{aligned} \tag{5.29}$$

together with the constraint (2.13) for the initial data U_0^a and V_0^a

$$(U_0(\sigma)\gamma_5 V_0(\sigma)) = -2i \,. \tag{5.30}$$

Note, that the τ-dependence of the dyads U^a and V^a can be removed by fixing the residual reparametrization symmetry by the choice $\varrho^\tau(\tau,\sigma) = \varrho_0^\tau(\sigma)$ or $\varrho^\tau(\tau,\sigma) =$ constant. As a result, U^a and V^a become integrals of motion and will be defined by the initial data of the variational problem in question. The representation (5.29) shows that the Goldstone fermion corresponding to partial spontaneous breaking of the $D = 4$, $N = 1$ global supersymmetry describes a static spinor configuration distributed along the closed superstring.

In view of (5.28) and (5.29) the remaining eqs. (5.19) and (5.20) take the form

$$\left(Y_{ab} U_0{}^b\right)^{\cdot} = 0 \,, \tag{5.31}$$

$$U_0^a Y'_{ab} U_0^b - 16 i \eta_0 \eta'_0 = 0 \,. \tag{5.32}$$

Equation (5.31) shows that the Majorana spinor Y_a defined by the relation

$$i(\gamma_5 Y)_a \equiv Y_{ab} U_0^b \,, \tag{5.33}$$

does not depend on the evolution parameter τ. We have used γ_5 in the definition of Y_a (5.33) to present it in the canonical form given by the Weyl representation (2.13)

$$Y_a = Y_{0a}(\sigma) \equiv \begin{pmatrix} y_{0\alpha}(\sigma) \\ \bar{y}_0^{\dot{\alpha}}(\sigma) \end{pmatrix}. \tag{5.34}$$

Then eq. (5.32) can be presented in the form of the τ independent constraint

$$i\left[(U_0\gamma_5 Y_0') - (U_0'\gamma_5 Y_0)\right] - 16i\eta_0\eta_0' = 0 \tag{5.35}$$

which is easily solved with respect to $Y_0(\sigma)$ using the expansion of $Y_{0a}'(\sigma)$ and $U_{0a}'(\sigma)$ in the dyad basis as it is given by the formula (3.2). So, the solution of the constraint (5.35) will fix the spinor $Y_{0a}(\sigma)$ as a function of the initial data $U_{0a}(\sigma)$, $V_{0a}(\sigma)$ and $U_{0a}'(\sigma)$ fixed by the statement of the variational problem under discussion. Note that the initial data U_{0a}' are equivalent to the specification of the initial data for the derivative coefficients in the $U_{0a}'(\sigma)$ expansion in the dyad basis. Therefore, one can consider the spinor $Y_{0a}(\sigma)$ as given. Then eqs. (5.33) are to be treated as equations for the restoration of the symmetric spinor matrix $Y_{ab}(\tau,\sigma)$

$$Y_{ab}(\tau,\sigma)U_0{}^b(\sigma) = i(\gamma_5 Y_0(\sigma))_a\,, \tag{5.36}$$

as a function of the given initial data $U_{0a}(\sigma)$ and $V_{0a}(\sigma)$.

The general solution of eq. (5.36) is presented by the sum

$$Y_{ab}(\tau,\sigma) = Y_{ab}^{(\mathrm{inho})}(\sigma) + Y_{ab}^{(ho)}(\tau,\sigma)\,, \tag{5.37}$$

where $Y_{ab}^{(\mathrm{inho})}(\sigma)$ is the solution of the inhomogenous equation

$$Y_{ab}^{(\mathrm{inho})}(\sigma) = \frac{i}{(U_0\gamma_5 Y_0)}(\gamma_5 Y_0(\sigma))_a(\gamma_5 Y_0(\sigma))_b \tag{5.38}$$

and $Y_{ab}^{(ho)}(\tau,\sigma)$ has to be the general solution of the homogeneous equation

$$Y_{ab}^{(ho)}(\tau,\sigma)U_0^b(\sigma) = 0\,. \tag{5.39}$$

So, we see that the total τ-dependence of the spin-tensor $Y_{ab}(\tau,\sigma)$ describing the superstring evolution in the enlarged superspace is concentrated in $Y_{ab}^{(ho)}(\tau,\sigma)$ and the static solution $Y_{oab}^{(\mathrm{inho})}(\sigma)$ may be treated as a domen-like vacuum solution.

To solve the homogeneous eq. (5.39) note that the relations

$$\chi_a^\Lambda(\tau,\sigma)U_0^a(\sigma) = 0\,, \qquad (\Lambda = 1,2,3) \tag{5.40}$$

are satisfied by the Majorana spinors $\chi_a^\Lambda(\tau,\sigma)$

$$\chi_a^\Lambda(\tau,\sigma) \equiv \{U_a, (\gamma_5 U_a), V_a\} \tag{5.41}$$

as it follows from the dyad definiton (2.12) and (2.13) and the solutions (5.28) and (5.29).

Taking into account the relations (5.40) we find the general solution of eq. (5.39)

$$Y_{ab}^{(ho)}(\tau,\sigma) = \sum_{\Lambda,\Sigma} F_{\Lambda\Sigma}(\tau,\sigma)\ \left[\chi_{0a}^\Lambda(\sigma)\chi_{0b}^\Sigma(\sigma) + \chi_{0b}^\Lambda(\sigma)\chi_{0a}^\Sigma(\sigma)\right], \tag{5.42}$$

where the τ-dependent factors $\sqrt{\varrho^\tau}$ and $1/\sqrt{\varrho^\tau}$ contained in $\chi_a^\Lambda(\tau,\sigma)$ have been removed to redefine the arbitrary functions $F_{\Lambda\Sigma}(\tau,\sigma)$ parametrizing $Y_{ab}^{(ho)}(\tau,\sigma)$.

Therefore, the general solution for the generalized coordinates $Y_{ab}(\tau,\sigma)$ (2.5) describing superstring evolution in the superspace enlarged by TCC coordinates is presented in the form

$$\begin{aligned} Y_{ab}(\tau,\sigma) &= \frac{i}{(U_0(\sigma)\gamma_5 Y_0(\sigma))}(\gamma_5 Y_0(\sigma))_a(\gamma_5 Y_0(\sigma))_b + \\ &\quad + \sum_{\Lambda,\Sigma} F_{\Lambda\Sigma}(\tau,\sigma)\ \left[\chi^{\Lambda}_{0a}(\sigma)\chi^{\Sigma}_{0b}(\sigma) + \chi^{\Lambda}_{0b}(\sigma)\chi^{\Sigma}_{0a}(\sigma)\right] \end{aligned} \tag{5.43}$$

with the total τ-dependence concentrated in the arbitrary scalar coefficients $F_{\Lambda\Sigma}(\tau,\sigma)$. This means that the superstring has no transverse oscillations, but generates longitudinal exitations propagating in the directions prescribed by the initial data for the dyads.

6. Superbranes with $3/4$ k-symmetry

The superstring action (5.4) is naturally generalized to the case of super p-brane by means of enlargement of the range of values for the world-sheet index μ

$$S_p = \frac{1}{2}\int d\tau d\sigma_1 \dots d\sigma_p\, \varrho^{\mu}\left(U_a W^{ab}_{\mu} U_b\right), \qquad \mu = (0, M), \qquad M = (1, 2, \dots, p), \tag{6.1}$$

where $\varrho^{\mu}(\tau,\sigma_M)$ is now a k-invariant world-volume density providing reparametrization invariance for the p-brane action S_p. The superstring equations of motion (5.6)–(5.9) will preserve their form (modulo the μ dimesionality extension) and can be exactly solved using the same method as for the two-dimensional eqs. (5.6)–(5.9). To solve the p-brane equations we choose the fermionic gauge condition (5.10) and extend the bosonic gauge condition (5.15) to

$$\varrho^{M}(\tau,\sigma_M) = 0, \qquad M = (1, 2, \dots, p). \tag{6.2}$$

Then, we get generalizations of the solutions (5.28) and (5.29)

$$\begin{aligned} \eta(\tau,\sigma_M) &= \frac{1}{\sqrt{\varrho^{\tau}}}\eta_0(\sigma_M), \\ U^a(\tau,\sigma_M) &= \frac{1}{\sqrt{\varrho^{\tau}}}U^a_0(\sigma_M), \\ V^a(\tau,\sigma_M) &= \sqrt{\varrho^{\tau}}\,V^a_0(\sigma_M), \\ \theta_a \equiv \theta_a(\sigma_M) &= -\eta_0(\sigma_M)(\gamma_5 V_0(\sigma_M))_a. \end{aligned} \tag{6.3}$$

and change the two remaining eqs. (5.31) and (5.32) by the following system of $(p+1)$ equations

$$\begin{aligned} \left(Y_{ab}U^b_0\right)^{\cdot} &= 0, \\ U^a_0\partial_M Y_{ab}U^b_0 - 16i\eta_0\partial_M\eta_0 &= 0. \end{aligned} \tag{6.4}$$

The substitution (5.33) used to introduce the Majorana spinor Y_a can be repeated again resulting in the τ-independence of Y_a

$$Y_a = Y_{0a}(\sigma_M) \equiv \begin{pmatrix} y_{0\alpha}(\sigma_M) \\ \bar{y}_0^{\dot\alpha}(\sigma_M) \end{pmatrix}. \tag{6.5}$$

So, eqs. (6.4) are transformed into the τ-independent system consisting of p constraints

$$i\left[(U_0\gamma_5\partial_M Y_0) - (\partial_M U_0\gamma_5 Y_0)\right] - 16i\eta_0\partial_M\eta_0 = 0 \tag{6.6}$$

which generalize the constraint (5.35). These constraints can be solved with respect to $Y_0(\sigma_M)$ using the expansion (3.2) for $\partial_M Y_{0a}(\sigma_N)$ and $\partial_M U_{0a}(\sigma_N)$ in the dyad basis. This solution is to be completed by investigation of the integrability conditions of these constraints. Modulo the integrability conditions the solution for the generalized coordinates $Y_{ab}(\tau,\sigma_M)$ (2.5) is presented in the form (5.43) with σ_M substituted instead of σ. Note that super p-branes with the dimensions $p = 3 = \dim(x\,\text{subspace}) - 1$, $p = 5 = \dim(z\,\text{subspace}) - 1$ and $p = 9 = \dim((x \bigoplus z)\,\text{space}) - 1$ may be a matter of additional interest. The solution (6.3) for $\theta_a(\sigma_M)$ presents the Goldstone fermion corresponding to the partial spontaneous breaking of the $D = 4$, $N = 1$ global supersymmetry and describes a static spin (or magnetic) configuration distributed along the closed p-brane. This solution can be treated as a p-dimensional generalization of the magnetic Nielsen-Olesen vortex discussed in the previous section.

7. Conclusion

We have considered a generalization of twistor-like approach to describe superstrings and super p-branes evolved in the superspace enlarged by the coordinates corresponding to the tensor central charges of the $D = 4$, $N = 1$ global supersymmetry algebra. A simple class of supersymmetric and κ-symmetric actions preserving 3/4 of the global supersymmetry has been proposed to prove the effect of the supersymmetry enhancement in the string/brane dynamics as a result of the inclusion of the TCC coordinates. These superstring (5.1) and super p-brane (6.1) actions are linear with respect to the diagonal matrix element W_U (3.14) of the supersymmetric Cartan one-forms W_{ab} (2.9) and generalize the actions of tensionless strings and branes. We have shown that the requirement for W_U to be an invariant of the κ-symmetry imposes one real condition on the κ-symmetry parameter. Alternatively, one could start choosing $W_{\gamma_5\varphi}$ to be invariant, but it will not change the conclusions. The requirement for another diagonal element to be an additional invariant of the κ-symmetry imposes one more real condition reducing the number of preserving supersymmetries to two instead of three ones. As a result, a super p-brane action including the diagonal elements W_U and W_V, e.g. the quadratic superstring action

$$S_{(U,V)} = g\int d\tau d\sigma\, \varepsilon^{\mu\nu}\left(U_a W_\mu^{ab} U_b\right)\left(V_a W_\nu^{ab} V_b\right), \tag{7.1}$$

will preserve 1/2 of the global supersymmetry. To restore the lost extra κ-symmetry it is necessary to add some compensating terms such as the Wess-Zumino terms or other differential two-forms constructed of the total set of the Cartan forms and/or their matrix elements. We intend to discuss this problem in another place.

A physical content of the studied models with enhanced κ-symmetry can be understood attracting the ideas advanced in [72, 73] and based on the conceptions developed in [70, 74]. Note also that in [79] string in the $D = 4$ space-time is described by a closed exactly solvable sector of the $\mathrm{SO}(1,1) \times \mathrm{O}(2)$ two-dimensional gauge model connected with the underlying gauge theory for the Nielsen-Olesen vortex.

We have shown that the presence of extra κ-symmetry in the proposed linear actions allows to find the general solutions for the Goldstone fermion and bosons corresponding to the partial spontaneous breaking of the global supersymmetry. We can try to use the Goldstone fermionic solution to describe a magnetic configuration associated with the Nielsen-Olesen vortex or with its p-dimensional generalization described by the long wave p-brane approximation. So, despite the fact that the fermions associated with supersymmetry do not carry electric charge they may describe magnetic properties of superstring and superbranes. This property of the Grassmannian coordinates θ_a was revealed in [80, 81], where a supersymmetric generalization of the Fokker-Schwarzschild-Tetrode-Wheeler-Feynman electromagnetic theory was studied. It was shown there that the spinors θ_a contribute to an anomalous magnetic moment (AMM) of neutral particles with spin 1/2. Also, it was shown in [82] that the contribution of the fixed AMM of $D = 4$, $N = 2$ charged massive superparticle interacting with extended Maxwell supermultiplet restores κ-symmetry broken by the minimal coupling [83].

Realization of such a possibility is supported by the following hint. As follows from the solution of the superstring and superbrane equations of motion, the effective spinors Y_a (5.34) and (6.5) are the first integrals

$$\dot{Y}_a = 0 \tag{7.2}$$

of these equations. Using this result one can propose to consider the preserving bilinear covariant

$$J_{MN} = iY_a(C\Sigma_{MN})^a{}_bY^b\,, \qquad \dot{J}_{MN} = 0\,, \tag{7.3}$$

where Σ_{MN} is the Lorentz group generators in the Majorana representation, to describe an effective spin/orbital momentum density of superstring/superbrane. Then the gauge invariant lagrangian density

$$L_{\mathrm{e.m.}} = \mu^* F^{MN} J_{MN}\,, \tag{7.4}$$

may be treated as an analog of electromagnetc interaction of spin 1/2 particles by means of their AMM, if μ^* has a sense of the phenomenological constant describing the value of the effective AMM of the superstring/superbrane.

Using the proposal [72] to consider cosmic supersymmetric Nielsen-Olesen vortices, it is interesting to treat the integrated AMM density of the vortex as a source of strong magnetic fields associated with cosmological objects and dark matter. Here we use an analogy with atomic spins of a crystal lattice as sources of magnetic fields in the domain walls of ferromagnetics. The lagrangian of spin waves treated as the Goldstone particles in magnetic media was constructed in [84, 85]. Applying this approach one can try to consider the Goldstone solutions associated with the p-vortices as standing spin waves distributed along the closed superstring or super p-brane.

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Of course, fixing exact physical contents of the considered model supposes carrying out its covariant quantization. According to the results which have been obtained here a suitable effective variable for the quantization in the fixed gauge (5.10) is the Majorana bispinor Y_a (see (5.33))

$$i(\gamma_5 Y)_a \equiv Y_{ab} U^b \,, \tag{7.5}$$

and the initial data $Y_{0a}(\sigma_M)$ and $U_{0a}(\sigma_M)$ appear to be proper primary variables for the canonical quantization in view of the solution (5.43). In terms of Y_a and η (5.11) the superstring (5.4) and super p-brane (6.1) actions are equivalenly presented as

$$S_p = \frac{i}{2} \int d\tau d\sigma_1 \ldots d\sigma_p \, \varrho^\mu \left\{ \left[(U\gamma_5 \partial_\mu Y) - (\partial_\mu U \gamma_5 Y) \right] - \eta \partial_\mu \eta + 2i \partial_\mu U^a \eta \theta_a \right\} . \tag{7.6}$$

The last term $\partial_\mu U^a \eta \theta_a$ in (7.6) is gauge dependent and it vanishes in the gauge (5.10). The restoration of the $\eta\theta_a$ term in the definition of Y_a (7.5) yields the new effective variable $\tilde{Y}_a$

$$(\gamma_5 Y)_a = \tilde{Y}_a - i\eta\theta_a \,. \tag{7.7}$$

The substitution of the shifted field $\tilde{Y}_a$ in (7.6) results in the new representation

$$S_p = \frac{i}{2} \int d\tau d\sigma_1 \ldots d\sigma_p \, \varrho^\mu \left\{ \left[\left(U^a \partial_\mu \tilde{Y}_a \right) - \left(\partial_\mu U^a \tilde{Y}_a \right) \right] - \eta \partial_\mu \eta \right\} \tag{7.8}$$

which includes only the κ-invariant Goldstone fermion η representing the whole fermionic sector of the super p-brane. The bosonic sector, originally presented by the primary bosonic variables Y_{ab}, or equivalently by x_m and z_{mn}, is encoded by $\tilde{Y}_a$. So, we conclude that the transformation of variables

$$Y_{ab} U^b = i\tilde{Y}_a + \frac{1}{2i} \left(U^b \theta_b \right) \theta_a \tag{7.9}$$

eliminates the correspondent gauge degree of freedom from the original actions (5.4) and (6.1) without using any gauge conditions. It shows a universal property of the extra κ-symmetry to remove not only the fermionic gauge degrees of freedom, but also the bosonic ones originally introduced by the coordinates x_m and z_{mn}.

The representation (7.8) generalizes the supertwistor formulations of superparticle action [50], [61]–[64] to the case of superstrings and superbranes, and includes eight bosonic fields described by the Majorana spinors $U_a(\tau, \sigma^M)$, $\tilde{Y}_a(\tau, \sigma^M)$ and one fermionic field described by the Grassmann variable $\eta(\tau, \sigma^M)$ which can be treated as the components of a real supertwistor. The property of the formulation (7.8) to include the gauge invariant variables (in the reparametrization gauge (6.2) and $\varrho^0(\tau, \sigma^M) =$ constant) suggests a realization of the gauge independent quantization in terms of the supertwistor components $(U_a, \tilde{Y}^a, \eta)$. In the twistor description of massless superparticle (see, e.g. [62]) this is a reason to consider the space of twistors as more fundamental than space-time. May be that point of view will give some advantages in the super p-brane theory. In any case, the quantization in terms of the supertwistor $(U_a, \tilde{Y}^a, \eta)$ have to be an effective way to find the spectrum of the super p-brane model with enhanced supersymmetry. Study of this problem is in progress now.

Acknowledgments

A. Z. would like to acknowledge Fysikum at the Stockholm University for kind hospitality. He is also grateful to Ingemar Bengtsson, Lars Bergström, Ulf Danielsson, Ulf Lindström and Hector Rubinstein for fruitful discussions. The work is partially supported by the grant of the Royal Swedish Academy of Sciences and Ukrainian SFFR project 02.07/276. A. Z. was partially supported by the grant of Axel Wenner-Gren Foundation and the Award CRDF-RP1-2108.

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